

# Optimal asset divestments with homogeneous products\*

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## Abstract

We study alternative market power mitigation measures in a homogenous goods industry where productive assets have asymmetric costs. We characterise the asset divestment by a dominant firm which achieves the greatest reduction in prices (taking the size of the divestment as given). The optimal divestment entails the sale of assets whose costs are close to the post-divestment price (i.e. they are price-setting). A divestment of this type can be several times more effective in reducing prices than divestments of low-cost assets. We also establish that virtual divestments (often employed in the power industry) are at best equivalent to low-cost divestments in terms of their impact on consumer welfare, and cannot replicate the optimal divestment.

JEL classification codes: D42, L13, L40, L94.

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## 1 Introduction

Regulatory and antitrust proceedings often require the application of remedies in the form of divestments, in order to mitigate market power or to prevent a reduction in competition from a merger. The appropriate choice of asset divestment often plays a critical role in ensuring that competition policy is effective. This paper studies the issue of optimal remedy design in a model of a homogenous goods industry where productive assets have different costs.

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The modelling framework that we use in this paper assumes that remedies are imposed on a dominant producer that faces a competitive fringe. Our analysis considers both the relative impact of different types of asset divestments, and the comparison between outright asset divestments and financial contracts (or “virtual” divestitures).<sup>1</sup> Outright divestments transfer productive assets from the dominant firm to the fringe. Virtual divestments instead allow third parties (e.g. traders) to exercise call options on the output of the dominant firm, obtaining it at fixed strike prices (in exchange for an option fee).

We find that the position of the divested capacity on the cost curve of the dominant firm has a strong effect on the impact of the divestment on market prices. The divestment policy which, for a given volume of divested capacity, achieves the greatest reduction in prices is denoted as the “optimal divestment” throughout this paper. Our results show that the optimal divestment includes capacity whose variable cost is intermediate. The location of the optimal divestment along the cost curve of the dominant firm is such that the divested capacity is withheld from the market in the pre-divestment equilibrium, but becomes price-setting post-divestment (meaning that its costs encompass the post-divestment price). In particular, access to the divested assets allows the fringe to bid more aggressively post-divestment, making the residual demand faced by the dominant firm flatter at the margin, and therefore increasing its incentives to lower prices and expand output. For sufficiently large divestments, the optimal divestment from the perspective of consumer welfare coincides with the socially efficient divestment. For yet larger divestments, the optimal remedy achieves the competitive price (unlike other types of divestments), thus achieving the first best in terms of both total and consumer welfare.

Divestment of low-cost assets are less effective than the optimal intervention because they involve non-strategic capacity which the dominant firm was already offering to the market pre-divestment. Their sale reduces the residual demand faced by the dominant firm, but at the same time increases its costs, resulting in a smaller price reduction relative to the optimal divestment. Divestment of high-cost capacity is also less effective since it weakens the competitive constraint which the fringe can exercise, relative to the optimal divestment. Overall the relationship between the location of the divested capacity on the cost curve of the dominant firm and the post-divestment price is therefore U-shaped.

The second main contribution of this paper is to compare outright asset divestments with virtual sales of capacity. We establish that a virtual divestment is less effective than the optimal outright divestment in reducing prices and that it can at best replicate the impact of a divestment of low-cost capacity (if the strike prices are set sufficiently low, implying that the virtual divestment acts like forward contract cover). This result implies that designing a virtual divestment so as to mimic the properties of the optimal asset divestment (i.e. setting strike prices equal to the variable costs of price-setting plants in the post-divestment equilibrium) does not ensure that the virtual divestment will be as effective as the corresponding physical divestment in reducing prices. Whilst the divestment of price-setting plants increases the pressure exercised by competitors to the dominant firm at the margin, the sale of financial contracts with strike prices close to the market price results in fewer of the options being exercised and therefore greater incentives for the dominant firm to increase prices (relative to a contract with lower strike prices).

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<sup>1</sup>Throughout the paper we refer to outright divestments of assets also as “physical divestments” or simply “divestments”.

Our framework and results are most directly applicable to the wholesale electricity industry, which is characterised by well-defined individual production facilities with different costs (i.e. power plants). In the electricity industry the divestment of physical and/or virtual capacity is often employed as a remedy by competition authorities and regulators to enhance competition. For example, outright plant divestments and Virtual Power Plant (VPP) schemes have been used across Europe in recent times, in the context of merger control proceedings, abuse of dominance investigations, and regulatory reviews of market power.<sup>2</sup>

The model that we use in this paper is also applicable to industries which share some of the essential features of electricity generation, most notably a homogenous final product and cost asymmetries between different assets. For example, the paper industry displays some of these characteristics, due to the different vintages of paper mills. The U.S. Department of Justice (DOJ) has recently ordered divestments in two cases involving the North American paper industry (Abitibi/Bowater, in 2007; and GPC/Altiivity, in 2008) in order to discourage capacity withholding by the merging parties. The issues raised in these decisions are related to those considered in this paper. Other industries where the framework used in this paper is of potential relevance include mining, other energy industries (e.g. gas), and homogenous products with high transport costs, where the difference in cost across production facilities is primarily determined by their distance from the main consumption centres (e.g. cement).

Yet more generally, the Horizontal Merger Guidelines issued by the U.S. DOJ and Federal Trade Commission (FTC) in August 2010 explicitly recognise the role played by different types of capacity in making output withholding profitable in markets involving homogenous products. These guidelines note that an output suppression strategy is more likely to be profitable after a merger if the margin on the suppressed output is relatively low, or if one of the merging firms has access to excess capacity at the pre-merger price (thus making the residual demand faced by the other party in the merger more elastic). Existing merger guidelines also discuss the role of contract cover and virtual divestitures in preventing or remedying unilateral effects.<sup>3</sup>

There is a relatively limited formal economic literature on the impact of divestments and virtual asset sales. This is often found in applications to the electricity generation market. Green (1996) is an early contribution which considers the impact of physical divestments in a model of supply-function equilibrium (including the case of divestments to a competitive fringe). This paper focuses on divestments that apply uniformly to the entire portfolio of an electricity generator, which can be modelled by simply changing the slope of its cost curve. This approach generates analytical convenient results, but cannot be used to analyse divestments of specific assets that are located at

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<sup>2</sup>Examples of European mergers or joint ventures in the electricity sector where divestments or VPPs have been required by the competition authorities include: Gas Natural/Union Fenosa (2009), EDF/British Energy (2008), Gas Natural/Endesa (2006), GDF/Suez (2006), Nuon/Reliant (2003), ESB/Statoil (2002) and EDF/EnBW (2000). Abuse of dominance cases where divestments of generation capacity or VPPs have been implemented as a remedy include proceedings involving E.On (2008), RWE (2008) and Enel (2006). Finally, divestments have also been used by regulators to mitigate market power of incumbent generators in the UK and Italy in the 1990s, whilst in Spain and Portugal VPPs have recently been employed to make the electricity market more competitive.

<sup>3</sup>The DOJ/FTC Horizontal Merger Guidelines of August 2010 note that unilateral effects will be more likely if a low share of output is committed for sale at prices unaffected by the conduct of the merging parties. The UK Competition Commission Guidelines of Merger Remedies of November 2008 explicitly discuss virtual divestitures as a possible remedy option.

different positions on the cost curve of a firm (which is the main focus of our paper).<sup>4</sup>

The literature has devoted more attention to the impact of forward contracts on market power. This issue is relevant to our analysis of virtual divestments since forward contracts can be interpreted as call options which are always exercised by the option holder, independently of the spot price. This strand of the literature includes the seminal contribution by Allaz and Vila (1993), which established that forward contracts can significantly increase competition in spot markets in a Cournot duopoly model.<sup>5</sup> Recent papers have noted that the pro-competitive impact of forward contracts in electricity markets may be reduced in the presence of repeated interaction (Schultz, 2009) or if contracts are not assigned to the largest firms in the market (de Frutos and Fabra, 2012). Our paper shows that even in the absence of these circumstances, contracts are significantly inferior to outright divestments as an instrument to increase competition. In related work, Willems (2006) compares the impact of financial and physical VPPs. Both types of intervention are modelled as financial instruments, whose effect is equivalent in a monopoly setting. We focus instead on the comparison of outright divestments with financial contracts, establishing a significant difference in their effectiveness also in a residual monopoly environment.

Our paper is also related to the competition case discussed by Armington *et al.* (2006) and Wolak and McRae (2008). These two articles describe in qualitative terms how divestments of capacity can be utilised to remedy the expected impact of a merger on prices, using the example of the proposed Exelon/PSEG electricity merger in the U.S. in 2006 (where the U.S. DOJ recommended the divestments of “ability” assets, whose cost was close to the market clearing price). Along similar lines, Crawford *et al.* (2007) report simulation results on the impact on prices of two types of divestment, in an oligopoly model with discrete volume bids calibrated on the British electricity market. They obtain that the sale of capacity with intermediate costs has roughly double the impact on price of the divestment of baseload (i.e. low-cost) assets, for a given level of demand. This quantitative result is in line with the theoretical results that we report below, and can be interpreted using our formal framework.

The structure of the remainder of this paper is as follows: Section 2 describes the set-up of our baseline model, including a characterisation of the equilibrium without remedies; Section 3 solves the case of divestments in the baseline model, presenting results on both prices and efficiency; Section 4 presents our results for virtual divestments, comparing them to those obtained for outright divestments in the baseline case; Section 5 generalises the baseline model, in order to illustrate the robustness of our key results to variations in some of our assumptions; and Section 6 concludes. The Appendix collects proofs and additional results not included in the main text.

## 2 Model set-up

### 2.1 The dominant firm with fringe assumption

As noted in the introduction, we model market power by assuming that only one firm (the dominant firm) acts strategically, and that all other producers behave as a competitive fringe that offers all

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<sup>4</sup>Vergé (2010) also considers the case of asset divestments that uniformly affect the cost schedule of the affected firm, in a Cournot framework.

<sup>5</sup>Newbery (1998), Green (1999), and Bushnell (2007) extend some of the results established by Allaz and Vila to competition in supply functions and Cournot competition with multiple firms.

of its output at cost. This assumption significantly simplifies the analysis, allowing us to model divestments in a flexible way (allowing for discontinuities in cost functions post-divestment) and yet obtain analytically tractable results. Whilst this assumption is stylised, it is a plausible representation of markets where there is a large incumbent and an unconcentrated group of smaller producers.<sup>6</sup> In this case, it is reasonable to assume that the smaller firms offer their output at cost (i.e. they collectively submit a supply function that coincides with their variable cost schedule), and that the dominant firm acts as a residual monopolist.<sup>7</sup>

More generally, our assumption is analytically equivalent to the one used in a bid-based approach to merger analysis (e.g. as described in Wolak and McRae, 2008, in connection with the U.S. DOJ’s analysis of the Exelon/PSEG merger). This approach models the impact of concentrations and divestments by assuming that the willingness to supply of the non-merging parties (which may or may not coincide with their costs) is constant pre- and post-merger.<sup>8</sup> A similar approach was used by the European Commission in its quantitative assessment of the merger between EDF and British Energy (and associated remedies) in late 2008, and is advocated as a screen for the impact of electricity mergers by Gilbert and Newbery (2008).

The assumption of a dominant firm facing a competitive fringe is also relevant to oligopoly models that assume that firms compete in discrete bids (as introduced by von der Fehr and Harbord, 1993, and empirically tested by Crawford *et al.*, 2007). In these models in any pure-strategy equilibrium only one firm acts strategically by withholding output relative to the competitive level, and all other firms produce as if they were bidding their output at cost. This implies that as long as a given pure-strategy equilibrium continues to exist after the strategic firm is subject to an asset divestment, such equilibrium can be broadly characterised using the results that we present in this paper.

## 2.2 The baseline model

In the baseline model we assume for simplicity that prior to a divestment the dominant firm and the fringe have the same linear and increasing marginal cost function, with slope  $\gamma$  (this symmetry assumption is relaxed in the more general case considered in Section 5). We define marginal costs for each firm  $i$  as  $c_i$  and output as  $q_i$ . We also adopt subscript  $d$  for the dominant firm and  $f$  for the fringe, so that  $c_i = \gamma q_i$  for  $i = d, f$ . We also assume in the baseline case that total demand is perfectly price inelastic and takes a constant value of  $\mu$  (this assumption too is generalised in the case considered in Section 5). We assume a constant willingness to pay for consumers that lies above the pre-divestment equilibrium price. This ensures that total and consumer surplus are finite.

We denote the equilibrium without any market power mitigation remedy (i.e. either an outright or virtual divestment) as the *pre-divestment* outcome. In this equilibrium, for a given spot price  $p$ , the competitive fringe always produces at its marginal cost. That is, we have  $p = c_f = \gamma q_f$ , where

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<sup>6</sup>This is the case in a number of European power markets, including Belgium, France, Ireland, Italy, and Portugal.

<sup>7</sup>Note that this equilibrium does not require the assumption that the dominant firm has a first-mover advantage in a sequential game. It will arise also in a simultaneous supply function game as long as all but one of the firms are sufficiently small and therefore face incentives to offer their output at (or close to) cost.

<sup>8</sup>The case where the bids of non-merging parties do not coincide with costs can be easily accommodated in a dominant firm with fringe set-up by changing the slope of the assumed bidding function of the fringe. This approach is conservative (i.e. it under-estimates the impact of a merger) if non-merging parties submit less aggressive bids post-merger, which is the case if one assumes competition in linear supply functions (as shown by Akgün, 2004). The fact that it is a conservative approach supports its use by competition authorities.

$q_f = \mu - q_d$ , implying that  $p = \gamma(\mu - q_d)$ . The dominant firm solves  $\max_p pq_d - \int c_d dq$ , which is equivalent to solving  $\max_{q_d} \gamma(\mu - q_d)q_d - (\gamma/2)(q_d)^2$ . The first-order condition<sup>9</sup> yields  $q_d^* = \frac{\mu}{3}$  and  $p^* = \frac{2}{3}\gamma\mu$  (where the latter denotes the pre-divestment price level). In the pre-divestment equilibrium the dominant firm therefore serves a third of demand, rather than half of demand as it would in a competitive equilibrium. The competitive price is given by  $p^c = \frac{1}{2}\gamma\mu$ .

### 3 Outright divestments in the baseline case

#### 3.1 Definition of an outright divestment

Our analysis of the impact of an asset divestment considers the sale of production assets (or plants) that are located contiguously on the marginal cost function of the dominant firm. The maximum output (or capacity) that can be produced by the divested units is defined as  $\delta$ . This parameter describes the *size* of the divestment. We treat  $\delta$  as an exogenous parameter. It can be interpreted as the outcome of interaction between a regulator that seeks to mitigate market power and other groups (including the dominant firm) which oppose such intervention. In the context of an antitrust procedure the size of the divestment can also be thought of as the smallest intervention required to eliminate the price increase that is associated with the competition concern.

We denote the highest marginal cost of the divested asset as  $\bar{c}$ . We use this parameter to define the *position* of the divestment on the cost curve of the dominant firm, and therefore the type of divestment that is being considered (e.g. if  $\bar{c}$  is low, the divestment includes low-cost or non-strategic capacity, whilst if it is close to the market price it includes price-setting capacity). For notational purposes we also define  $q'$  as follows:  $q' = \frac{\bar{c}}{\gamma}$ . Divested capacity is transferred to the fringe through a one-off competitive auction process that we do not model. In the post-divestment equilibrium the divested assets are therefore offered to the market at cost.

A divestment affects both the marginal cost and residual demand functions of the dominant firm, as it is illustrated in Figure 1:

- It increases the cost function of the dominant firm above a given marginal cost level (i.e. for  $c_d > \bar{c} - \gamma\delta$ ). We refer to this as a *cost-increasing* effect. This effect is relevant to prices in the post-divestment equilibrium if the cost of the divested capacity is sufficiently low (implying that the dominant firm is utilising at least part of the divested capacity in the pre-divestment equilibrium). The presence of the cost-increasing effect tends to *reduce* the pro-competitive impact of a divestment *ceteris paribus* because it induces the dominant firm to set higher prices, since its costs are higher.
- A divestment also changes the residual demand curve of the dominant firm, in two ways: it introduces a flatter segment for prices that are between the lowest and highest cost of the divested capacity (i.e.  $p \in (\bar{c} - \gamma\delta, \bar{c})$ ); and it displaces the residual demand function downwards by the size of the divestment  $\delta$  for sufficiently high price levels (i.e. for  $p \geq \bar{c}$ ). We denote the first demand effect a *demand-slope* effect, and the second demand effect a *demand-shift* effect.

We restrict our analysis to divestments whose size (relative to demand) lies strictly above a lower bound denoted as  $\delta^L$ , which takes a value of  $\left(1 - \frac{12}{5\sqrt{6}}\right)\mu \approx 0.02\mu$ ; and at or below an upper bound

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<sup>9</sup>The second-order conditions are satisfied throughout the analysis.

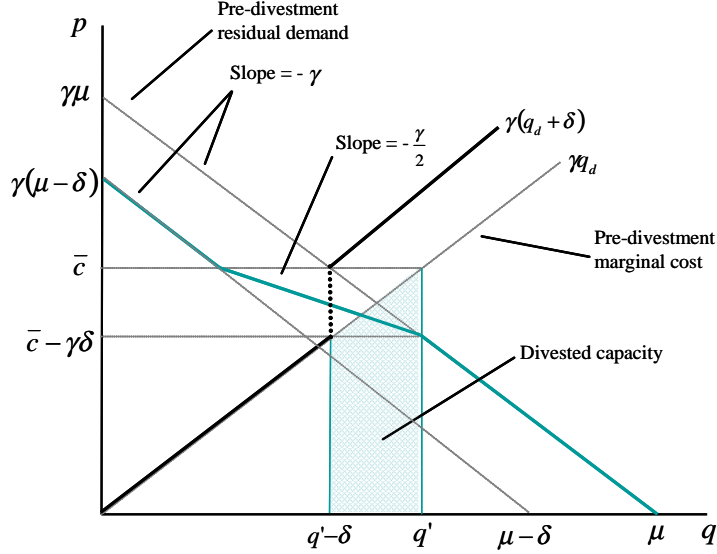


Figure 1: Description of the model set-up and of the impact of a divestment.

$\delta^H$ , which equals  $\left(1 - \frac{2}{\sqrt{6}}\right) \mu \approx 0.18\mu$ . These bounds on the size of the divestment follow from the formal derivation of the post-divestment price function that we illustrate below, and define the cases for which this function holds. This is a relative wide range for the size of a divestment, which captures realistic scenarios.<sup>10</sup> In the rest of this section we first focus on the impact on market prices (and therefore consumer welfare) of divestments, and then discuss their effects on efficiency and profits.

### 3.2 The post-divestment price function

As mentioned in the introduction, in this paper we refer to the divestment which achieves the lowest level of prices (for a given size  $\delta$ ) as the optimal divestment. This does not necessarily correspond to the socially optimal intervention, as it is shown below. To identify the location of the optimal divestment (for a given  $\delta$  and  $\mu$ ), we derive the unique equilibrium level of the post-divestment price for each value of  $\bar{c}$ . We denote this post-divestment price function as  $p(\bar{c})$ . This function is formally derived in the Proof of Proposition 1 below (contained in Appendix A.2). It is also plotted in Figure 2, and summarised in Table 1 in Appendix A.1.

As Figure 2 illustrates, the post-divestment price function is U-shaped, implying that the optimal divestment is located at an intermediate position in the cost function of the dominant firm. We define six distinct segments in the post-divestment price function (denoted from  $I$  to  $VI$ ), corresponding to different intersections of the marginal revenue schedule of the dominant firm with its marginal cost, as the position of the divestment (i.e.  $\bar{c}$ ) varies. The different levels of the equilibrium price result from the fact that post-divestment the marginal cost of the dominant firm is discontinuous at  $c = \bar{c} - \gamma\delta$ , and its residual demand function has two kinks (at  $\bar{c}$  and at  $\bar{c} - \gamma\delta$ ), which in turn results in discontinuities in the marginal revenue function. The presence of these discontinuities

<sup>10</sup>The working paper version of this article (Federico and López, 2009) also studies the cases of divestments that are smaller or larger than the range considered here. The corresponding results are reported below.

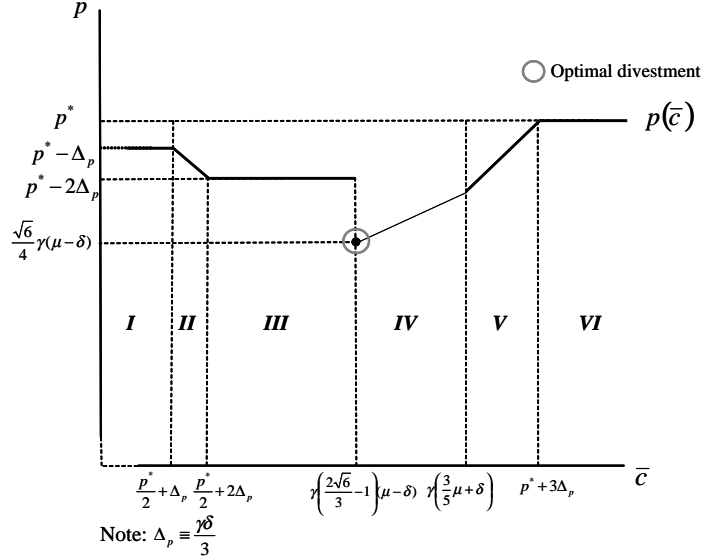


Figure 2: The post-divestment price, as a function of the position of the divested capacity.

complicates the analysis of the optimal divestment, generating different values of the equilibrium price as  $\bar{c}$  varies. Nonetheless we are able to obtain a tractable closed form solution for the post-divestment price function and for the optimal divestment, given the simplifying assumptions adopted in our baseline model. Before discussing the main features of the optimal divestment (in the next sub-section), we briefly describe the properties of the various segments of the post-divestment price function, in order to be able to illustrate the reason why the optimal divestment is found at an intermediate position in the cost function of the dominant firm.

The three segments of the post-divestment price function where an interior equilibrium exists *and* a divestment lowers the price (these are segments *I*, *III* and *IV*) differ depending on which of the three potential effects of a divestment on the cost and demand schedules of the dominant firm (as described above) are present:

- Segment *I* describes the case of *low-cost* or (non-strategic) asset divestments. A low-cost divestment is one that includes efficient capacity that the dominant firm would have utilised absent the divestment, even at the lower post-divestment price. In the case of a low-cost divestment, the cost-increasing and the demand-shift effects both apply. This leads to a price reduction since the second effect outweighs the first. The price that is obtained by a low-cost divestment is constant, and equals  $p(\bar{c}) = p^* - \Delta_p$ , where  $\Delta_p \equiv \frac{\gamma\delta}{3}$ .
- In segment *III*, the demand-shift effect still applies, but the cost-increasing one no longer does, since the divested capacity includes more expensive assets which the dominant firm would have not utilised in the post-divestment equilibrium even if they had been available. The divested capacity is however sufficiently competitive to be fully utilised by the competitive fringe at the post-divestment price, thus reducing the residual demand faced by the dominant firm by  $\delta$ . The price reduction from the divestment is therefore larger than in segment *I* (more precisely, it is twice as large) since the cost-increasing effect is absent but the same demand-reducing



effect is present.

- In segment *IV*, the demand-slope effect applies instead of the demand-shift effect since the cost of the divested units is sufficiently high so as to induce the dominant firm to price on the flatter segment of its residual demand curve, thus ensuring that some of the divested capacity does not produce post-divestment. The optimal divestment is located at the lower bound of this segment. Prices in segment *IV* are always below those in segment *III* if the divestment is of sufficiently small size (i.e. for  $\delta < \frac{\mu}{10}$ ). For larger values of  $\delta$  (i.e. for  $\delta \geq \frac{\mu}{10}$ ) prices in segment *IV* are below those in segment *III* only if  $\bar{c}$  is sufficiently low. As we shall see in Section 3.4.1, whether  $\delta$  is at/above or below the threshold value  $\frac{\mu}{10}$  plays an important role in the determination of the socially optimal divestment (Proposition 2).

The reason why the equilibrium price jumps downwards between segments *III* and *IV* is that the marginal revenue function jumps upwards at the first kink in the residual demand of the dominant firm, intersecting twice with the marginal cost function (the two intersections correspond to segments *III* and *IV* respectively). Within the relevant range of  $\bar{c}$  for which these two solutions are possible, the dominant firm will select the outcome associated with higher profits. At the optimal divestment (whose highest cost is denoted as  $\bar{c} = \hat{c}$ ), equilibrium profits are the same in the two segments, and thereafter (for  $\bar{c} > \hat{c}$ ) they are higher in segment *IV*. In order to move from segment *III* to segment *IV*, the dominant firm must lower its price discontinuously, so as to price on the flatter part of its residual demand schedule.

The other two segments of the post-divestment price function which yield a reduction in prices relative to the pre-divestment equilibrium (i.e. segments *II* and *V*) represent corner solutions, where the dominant firm finds it optimal to produce at output levels at which the marginal cost and revenue functions respectively are discontinuous:

- Segment *II* is defined by the intersection of the marginal revenue schedule with the marginal cost function at the point where the latter jumps upwards. In segment *II* the dominant firm therefore selects an output level that is exactly equal to  $q' - \delta$  (i.e. the quantity corresponding to most efficient unit of capacity that is divested), and does not utilise any of the capacity that is more expensive than the divested assets.
- Segment *V* is defined by the intersection of the marginal cost function with the marginal revenue function at the point where the latter jumps downwards (due to the second kink in the residual demand function). In segment *V* the dominant firm therefore prices on the second kink of its residual demand curve, implying that the price is equal to the lowest cost of the divested capacity (i.e.  $p = \bar{c} - \gamma\delta$ ) and that none of the divested assets produce. Prices remain at this level as  $\bar{c}$  increases further until the divestment no longer represents a constraint on prices (i.e. until the point where the lowest cost of the divested capacity equals the pre-divestment price, corresponding to the lower bound of segment *VI*).

### 3.3 The optimal divestment

The post-divestment price schedule described above indicates that the lowest post-divestment price is located at the lower bound of segment *IV*, at the point where the post-divestment price function is discontinuous in  $\bar{c}$ . The following Proposition characterises the optimal divestment.

**Proposition 1 (Optimal divestment)** *The optimal divestment is obtained by setting  $\bar{c} = \hat{c} \equiv \gamma \left( \frac{2\sqrt{6}}{3} - 1 \right) (\mu - \delta)$ . This divestment has the following key features:*

- *it includes only capacity that the dominant firm withholds from the market in the pre-divestment equilibrium;*
- *it is located between the competitive and the pre-divestment price, that is:  $\hat{c} \in (p^c, p^*)$ ; and*
- *it induces the dominant firm to price on the flatter segment of its post-divestment residual demand function, implying that the cost range of the divestment encompasses the post-divestment price, i.e.  $p(\hat{c}) \in (\hat{c} - \gamma\delta, \hat{c})$ .*

*The price which results with the optimal divestment is given by  $p(\hat{c}) = \frac{\sqrt{6}}{4}\gamma(\mu - \delta)$ . For  $\delta = \delta^H$  we have that  $p(\hat{c}) = p^c$ , otherwise  $p(\hat{c}) \in (p^c, p^*)$ .*

Proposition 1 establishes that the optimal divestment includes capacity that the dominant firm is withholding from the market in the pre-divestment equilibrium, and that becomes price-setting post-divestment. The divestment leads to the largest reduction in prices for two related reasons: (a) it does not include capacity that the generator was using pre-divestment, and it therefore does not increase its cost relative to the pre-divestment equilibrium; and (b) it ensures that at the margin the dominant firm faces a flatter residual demand curve (relative to the pre-divestment situation). The latter effect induces the firm to drop its price in order to capture more output from the competitive fringe, and prevent some of the divested capacity from producing. Only at the lower bound of the range of  $\delta$  that we consider (i.e. for  $\delta = \delta^L$ ), the optimal divestment is such that none of the divested capacity produces and the price therefore equals the lowest cost of the divested capacity (i.e.  $p(\hat{c}(\mu)) = \hat{c}(\mu) - \gamma\delta^L$ ).<sup>11</sup>

As described in Proposition 1, the cost of the optimal asset divestment takes an intermediate value. The highest cost of the optimally divested capacity is below the pre-divestment price, meaning that the divestment needs to be sufficiently competitive to be effective. The optimal divestment however cannot be too competitive. In particular, its highest cost needs to lie above both the competitive price  $p^c$  (as stated in the Proposition), and the highest cost of the most efficient capacity of size  $\delta$  not utilised by the dominant firm in the pre-divestment equilibrium (i.e.  $\hat{c} > \gamma \left( \frac{\mu}{3} + \delta \right)$ ). Only if the size of the divestment is sufficiently large (i.e. at  $\delta = \delta^H$ ) the optimal divestment corresponds to the lowest-cost capacity of size  $\delta$  that is withheld by the dominant firm pre-divestment. In this case, the optimal divestment also achieves the competitive price.<sup>12</sup>

Cheaper divestments than the optimal divestment have a lower pro-competitive effect because the divested assets are “too efficient”. This means that the price which the dominant firm would need to set to prevent some of the divested capacity from producing is too low. The dominant firm therefore finds it optimal to set a higher price and suffer a larger reduction of its residual demand (as in segment *III* of the post-divestment price function). As the cost of the divested capacity falls further (as in segment *I*) divestments become even less effective, since they involve capacity that the dominant firm was already using pre-divestment. The sale of this capacity increases the cost of the dominant firm, further reducing its incentives to drop its prices.

<sup>11</sup>In Federico and López (2009) we show that this result also holds for all values of  $\delta$  less than  $\delta^L$ .

<sup>12</sup>These results also extend to  $\delta \in (\delta^H, \frac{\mu}{2})$ , as shown in Federico and López (2009).

More expensive divestments than the optimal divestment are also less effective because they involve inefficient capacity which exercises a weaker constraint on the dominant firm. If the cost of the divested assets are too high, the divestment is completely ineffective in reducing prices (as in segment *VI* of the post-divestment price function).

The difference in the price impact of different divestments can be significant, as we illustrate in Section 4 of the paper in the discussion of virtual divestments (which are at best equivalent to divestments of low-cost assets, as it is shown below).

Our results on the location and effects of the optimal divestment depend on the combination of straightforward economic effects on the pricing incentives faced by the dominant firm, namely the absence of an increase in its costs due to the asset sale, and the fact that the costs of the divestment are such that the dominant firm face a flatter residual demand at the margin. These effects are robust to variations in the simplifying assumptions adopted in our baseline model, including the fact that the marginal cost schedule of the dominant firm and of the fringe is symmetric, and that demand is price inelastic. This is illustrated in Section 5, by reference to a more general model of divestments.

### 3.4 Impact of outright divestments on efficiency and on profits

Our analysis so far has centered on the impact of divestments on consumer surplus, since regulators and competition authorities typically focus on this welfare measure. In this section of the paper, we extend the results of our baseline model also to total welfare and industry profits.

#### 3.4.1 Efficiency

Our assumption of perfectly inelastic demand implies that in the baseline case divestments increase aggregate welfare if they reduce the total costs of producing the fixed level of output  $\mu$ . A divestment can affect total costs through three distinct output effects, relative to the pre-divestment equilibrium: (i) a reduction in the output of high-cost capacity owned by the fringe; (ii) a change in the output of the divested capacity; and (iii) a change in the net output of the dominant firm (i.e. its output net of any fraction of the divested capacity which was being utilised by the dominant firm in the pre-divestment equilibrium).<sup>13</sup>

The following Proposition summarises the properties of the socially optimal divestment.

**Proposition 2 (Welfare)** *There is a threshold value of  $\delta$  (defined as  $\delta^W < \delta^H$ ) above which the optimal divestment from the point of view of consumer surplus coincides with the socially optimal divestment. For lower values of  $\delta$ , the costs of the socially optimal divestment can be either higher or lower than the costs of the optimal divestment from the perspective of consumer welfare. In this case, the socially optimal divestment is located either at the lower bound of segment *III* (for  $\delta \in [\frac{\mu}{10}, \delta^W)$ ), or at the lower bound of segment *V* (for  $\delta < \frac{\mu}{10}$ ), depending on which of these two types of divestment yields the lower price.*

We have already established in Proposition 1 that the optimal divestment delivers the social first-best (i.e. a competitive price) if the divestment is sufficiently large (i.e.  $\delta = \delta^H$ ). Proposition

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<sup>13</sup>The assumption of increasing marginal costs implies that a sufficient condition for price-reducing divestments to be welfare-increasing is that the net output of the dominant firm does not decrease post-divestment. This condition is satisfied in all segments of the post-divestment price function but for segment *III*. The welfare effects of a divestment located in this segment are ambiguous, as it is shown in Federico and López (2009).

2 extends this result by showing that as long as the divestment size is sufficiently close to  $\delta^H$ , the optimal intervention from the perspective of social and consumer welfare coincide. In the baseline case we obtain that this is the case for  $\delta \geq \delta^W \approx 0.16\mu$ .

For lower values of  $\delta$  however the optimal divestment in terms of consumer welfare does not maximise efficiency because it leads to relatively high-cost divested capacity producing in equilibrium. Productive efficiency is enhanced either if more expensive capacity is divested, so that none of it produces in equilibrium, whilst still achieving a price reduction (as at the lower bound of segment *V* of the post-divestment price function); or if instead cheaper capacity is sold, implying that all the divested assets produce post-divestment, whilst ensuring that the net output of the dominant firm does not fall (as at the lower bound of segment *III*). The relative impact on efficiency between these two alternatives is determined by their respective impact on price. This is in turn a function of whether the size of the divestment is greater than the threshold value  $\frac{\mu}{10}$  (as set out in Section 3.2). For  $\delta$  sufficiently low (i.e.  $\delta < \frac{\mu}{10}$ ) we obtain that a divestment located at the lower bound of segment *V* (i.e. involving high-cost assets) leads to lower price than in segment *III* and is therefore the socially optimal intervention. For intermediate values of  $\delta$  (i.e.  $\delta \in [\frac{\mu}{10}, \delta^W)$ ) we obtain instead that selling lower cost assets located at the lower bound of segment *III* results in lower prices than in segment *V* and a more efficient outcome. Our results also imply that the divestment of low-cost non-strategic assets (located in segment *I*) never represents the most socially efficient measure.

### 3.4.2 Profits

We can derive the impact of a divestment on industry profits from our results on prices and consumer welfare. In doing so, we also consider the impact of divestments on the distribution of profits between the dominant firm and the competitive fringe. In order to study this issue, we assume that divested assets are sold to the fringe through a competitive tender process, so that the dominant firm receives the profits earned by the divested assets at the post-divestment prices and quantities. This assumption broadly reflects how divestments are implemented in practice in the context of antitrust cases. Our results on the impact of divestments on profits are summarised in the corollary below (with additional results contained in Appendix A.4).

**Corollary 1** *Any price-reducing divestment leads to a fall in industry profits. Both the dominant firm and the competitive fringe prefer a high-cost divestment with a limited price impact, if the cost of the divested capacity is above a threshold value located within segment *V* of the post-divestment price function sufficiently high (i.e. for  $\bar{c} > \frac{2}{3}\gamma(\mu + \delta)$ ). For values of  $\bar{c}$  lower than this threshold value, then the preferred divestment from the point of view of industry profits includes instead low-cost assets located in segment *I* of the post-divestment price function.*

This Corollary indicates that aggregate industry profits fall following a divestment that includes sufficiently competitive assets and which therefore is effective in reducing prices. Moreover, under our assumption that the divested capacity is sold by the dominant firm to the fringe through a competitive auction, then the preferences of the dominant firm and of the competitive fringe with respect to the type of divestment are generally aligned.

The fringe always suffers from a reduction in price due to a divestment, since this leads to a loss of both infra-marginal profits and of profits on the capacity displaced by the divestment. The losses incurred by the fringe are proportional to the price effect of divestment. This means that

for an intervention of a given size, the competitive fringe prefers asset divestments whose costs are sufficiently high and that are therefore ineffective. If more competitive divestments are selected instead, then the competitive fringe gains from the choice of efficient but non-strategic assets, relative to less competitive capacity.

The dominant firm too always loses out from a divestment of its assets, by construction (otherwise, it would be able to realise any profit gain also pre-divestment). In order to reduce the profit loss from a divestment, the dominant firm too prefers the sale of assets associated with a relatively limited impact on prices (i.e. divestments located in segments  $I$  and  $V$ ). For this type of divestments, the losses suffered by the dominant firm are proportional to the price effect of the divestment, implying that preference of the dominant firm on the position of the divested capacity are aligned with that of the fringe. These results indicate that a “coalition” of the dominant firm and its competitors may lobby for types of divestments that do not benefit consumers.

## 4 Virtual divestments in the baseline case

In this section we describe the impact of virtual divestments on prices and welfare in the baseline model. Virtual divestments in the form of VPP schemes have been used by regulators and competition authorities in a number of European electricity markets to reduce effective concentration. Such schemes are often structured as a set of contract obligations on some electricity producers, whereby these producers must pay to the holders of the contracts any positive difference between the spot price and the contract strike price, for the quantity specified in the contract (like in a one-way call option). The quantity and strike prices associated with the contracts are typically exogenous, and set by a regulator. Virtual divestments can include different type of contracts, depending on the strike prices that are chosen, and the time periods during which the options can be exercised.<sup>14</sup> The option contracts are typically sold to the market in periodic auctions, which determine the option fee that is payable to producers for the contract(s). Our modelling of virtual divestments is directly relevant to the use of VPPs in electricity markets or energy release programs in gas markets<sup>15</sup>, but also applies to the impact of financial contracts relative to outright divestments which is a question of broader interest.

In what follows we model virtual divestments flexibly, allowing option contracts to have different strike prices. For example, the strike prices of the virtual divestment may be set so as to mimic the actual cost structure of the firm that is subject to the contract. This however does not have to be the case, and our set-up allows us to also consider simpler contracts (e.g. a group of call options with a constant strike price), or more complex ones (with strike prices that do not necessarily correspond to production costs).

We abstract from some of potential institutional advantages associated with virtual divestments relative to divestments (e.g. relating to ease of implementation and reversibility). The formal results that we present in what follows can be therefore interpreted as providing an illustration of the drawback associated with virtual divestments in terms of weaker market power mitigation, and

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<sup>14</sup>For example in Spain both baseload and peak VPPs have been imposed on incumbent generators between 2007 and 2010. The baseload VPP applied to all hours of a given period, whilst the peak VPP could only be exercised on week-days, between 8 am and midnight. The baseload VPP had a lower strike price than the peak VPP.

<sup>15</sup>For a survey of the application of VPPs in the energy sector see Ausubel and Cramton (2010).

of the corresponding institutional advantages that should be associated with VPPs to justify their adoption.

We formally define a virtual divestment as a set of one or more one-way call option contracts. Each option contract  $j$  is defined by the pair  $(\delta_j, f_j)$  and commits the producer subject to the virtual divestment to pay any positive difference between the market price  $p$  and an exogenously-set strike price  $f_j$  to the holder of the option, for the quantity  $\delta_j$  specified in the contract. An option is exercised (i.e. the holder demands a payment from the producer) only if  $p > f_j$ .

Suppose that there exist  $n \geq 1$  contracts. Letting  $\Theta$  be the set of the  $n$  contracts ordered from the lowest to the highest strike price, we have that  $\Theta = \{(f_1, \delta_1), (f_2, \delta_2), \dots, (f_n, \delta_n)\}$ , with  $f_1 \leq f_2 \leq \dots \leq f_n$ . By definition, if  $p > f_n$  the  $n$  options will be exercised. If instead  $p \leq f_n$ , only a subset of options will be exercised.

The equilibrium spot price for a given virtual divestment  $\Theta$  is defined as  $p^\nu(\Theta, \delta)$ , where  $\delta = \sum_{j=1}^n \delta_j$ . As for the case of outright divestments, we denote  $\delta$  as the *size* of the virtual divestment. Note that if all the options are exercised (i.e.  $f_n < p^\nu(\Theta, \delta)$ ), the virtual divestment acts like a forward contract of size  $\delta$ .

#### 4.1 Prices with virtual divestments

The following Proposition describes the impact of a virtual divestment on prices in the baseline model.

**Proposition 3 (Virtual divestments)** *The largest price reduction achieved by a virtual divestment of size  $\delta$  equals  $\Delta_p \equiv \frac{\gamma\delta}{3}$ . This is the same impact on price as the one achieved by a low-cost outright divestment of size  $\delta$ . This price effect is achieved if the highest strike price in the virtual divestment scheme lies below the post-divestment price (i.e.  $f_n < p^* - \Delta_p$ ). This implies that all the options in the virtual divestment are exercised, so that it acts like a forward contract of size  $\delta$ .*

The reason why a virtual divestment whose options are all exercised leads to a reduction in prices is well-known from the existing literature on forward contracts. Prices fall when a forward contract is imposed on a dominant producer due to the fact that the contract “sterilises” part of the revenues of the dominant firm, making them independent of the market price. Formally, this leads to an outwards shift in the marginal revenue function of the dominant firm, due to the fact that less of its infra-marginal output receives the market price, so that a price reduction becomes less costly for the firm. This in turn leads to an output increase for the dominant firm, along its pre-divestment marginal cost function, and consequently a price reduction.

A forward contract yields the same price as a low-cost outright divestment of the same size because it effectively removes an amount  $\delta$  from the infra-marginal capacity of the dominant firm. It is as if the dominant firm “reserves” this part of its infra-marginal assets to meet its contract obligations, foregoing the market profits earned by this capacity. This is equivalent to simply not owning  $\delta$  units of capacity and not being subject to the contract obligation, as in the case of a low-cost divestment of size  $\delta$  to a competitive fringe.

Following essentially the same reasoning, it can be shown that the equivalence between the divestment of low-cost assets to a competitive fringe and a forward contract of the same size also holds in oligopoly models (e.g. in Cournot, or a linear supply function model such as the one

considered by Green, 1999). This equivalence relies on the assumption that divested assets are transferred to a firm with no or insignificant market power (which therefore offers the assets at cost, without withholding any other capacity that it may own). If instead the low-cost assets are transferred to a firm with some market power, then a divestment will be less effective in moderating prices relative to a forward contract (assuming that the financial instrument is held by firms with no physical assets, e.g. traders). This will not necessarily be the case for divestments of price-setting assets, on the basis of the results on outright divestments presented above.

The impact of a virtual divestment does not depend on the distribution of strike prices, as long as the highest strike price set in the contract  $f_n$  lies below  $p^* - \Delta_p$ . When some strike prices lie above  $p^* - \Delta_p$ , then it is not profitable to exercise some of the options in the virtual divestment. This in turn results in a larger share of the output for the dominant firm benefitting from the market price, inducing it to set a higher price (relative to a forward contract). The optimal virtual divestment from the perspective of maximising consumer welfare corresponds therefore to a set of call options with low strike prices, acting like a forward contract.

## 4.2 Comparison between the virtual divestments and outright divestments

### 4.2.1 Prices

Proposition 3 implies that optimal virtual divestments are never more effective than optimal physical divestments in reducing prices. To explicitly compare the maximum price impact achieved by both interventions we define a function  $R\left(\frac{\delta}{\mu}\right)$ , which measures the ratio between the price reductions achieved by the optimal divestment and by the optimal virtual divestment (that is,  $R = \frac{p^* - p(\hat{c})}{\Delta_p}$ ). Given that the price reduction achieved by the optimal virtual divestment is proportional to  $\delta$ , the function  $R$  also describes the size of the virtual divestment required to match the price reduction achieved by an optimal divestment of size  $\delta$ , expressed as a ratio of  $\mu$ .

From the results obtained so far we can derive that  $R = \frac{3\sqrt{6}}{4} + \left(2 - \frac{3\sqrt{6}}{4}\right) \frac{\mu}{\delta}$ . This function is decreasing in  $\frac{\delta}{\mu}$ , and ranges between 9.9 and 2.7 for  $\delta \in (\delta^L, \delta^H]$ . Exploiting the results for  $\delta \leq \delta^L$  and  $\delta \in (\delta^H, \frac{\mu}{2})$  that are derived in Federico and López (2009)<sup>16</sup> we can also show that  $R$  is continuous and decreasing for all values of  $\delta$  between 0 and  $\frac{\mu}{2}$ , and that it equals 1 for  $\delta = \frac{\mu}{2}$ . The latter result indicates that a virtual divestment is as effective as the optimal outright divestment only in the case of very large asset sales (equivalent to the competitive output level of the dominant firm).

The results captured by the function  $R$  shows that there is a potentially significant quantitative difference in the effectiveness of outright and virtual divestments as a market power mitigation measure. The reason for this difference is that a virtual divestment only affects the composition of the revenues obtained by the dominant firm, but cannot be used to increase the production capacity that is available to competitors of the dominant producer, in particular by targeting assets that the dominant firm would otherwise withhold from the market. Unlike the optimal outright divestment, a virtual divestment cannot increase the competitive pressure faced by a dominant firm at the margin, by increasing the output available to competitors. A further implication of this result is that mimicking the properties of the optimal divestment by setting a range for the strike prices in the virtual divestment that is similar to the cost range of the optimal divestment does not increase

<sup>16</sup>For  $\delta \in (0, \delta^L]$  we have that  $R\left(\frac{\delta}{\mu}\right) = \sqrt{2\frac{\mu}{\delta} - 1}$ . For  $\delta \in (\delta^H, \frac{\mu}{2}]$  we have that  $R\left(\frac{\delta}{\mu}\right) = \frac{1}{2} \frac{\mu}{\delta}$ .

the pro-competitive impact of the virtual divestment.<sup>17</sup>

### 4.2.2 Efficiency and profits

The optimal virtual divestment is welfare-increasing, since it induces the dominant firm to increase its output, thus leading to a reduction in the output of high-cost capacity belonging to the competitive fringe. However, the optimal outright divestment always leads to a greater efficiency increase than the optimal virtual contract, as stated in the following Proposition.

**Proposition 4** *Optimal divestments increase total welfare by more than the optimal virtual divestment of the same size.*

This Proposition implies that outright divestments, if chosen optimally, can increase both consumer welfare and efficiency by more than virtual divestments. The intuition for the efficiency result is that at the optimal divestment a greater fraction of the production of the competitive fringe shifts from high-cost assets to lower cost capacity (i.e. that which is divested), coupled with the fact that the dominant firm also increases its net output (for  $\delta < \delta^H$ ). This efficient reallocation of output takes place to a greater extent than with a virtual divestment, since the latter yields a lower reduction in prices. These effects outweigh the inefficiency associated with production by relative expensive divested capacity under the optimal outright divestment.

Our earlier results on the impact of profits of an outright divestment also imply that producers will jointly and individually prefer a virtual divestment to a physical one, unless the costs of the assets chosen to be divested are sufficiently high.

## 5 Robustness of results

The results presented so far on the impact of different types of divestments on price (and the related results on virtual divestments) rely on standard economic effects. These effects can naturally be expected to apply also in a more general model than the baseline framework that we employed above for expositional clarity.

In order to generalise our baseline model we assume that the cost schedules of the dominant firm and the fringe can have different slopes, denoted as  $\gamma_i$  for  $i = d, f$ . We also assume that the overall demand schedule is price-elastic, with slope  $\rho$ , so that  $q_d + q_f = \mu - \rho p$ . It is straightforward to show that in such a general model the pre-divestment price  $p^*$  equals  $\frac{1+\gamma_d x}{2+\gamma_d x} \frac{\mu}{x}$  (where  $x = \rho + \frac{1}{\gamma_f}$ ), and that the pre-divestment output of the dominant firm  $q_d^*$  equals  $\frac{\mu}{2+\gamma_d x}$ .

Following the same methodology used for the baseline model, we can identify a unique optimal divestment, for an intermediate range of the size of the divestment  $\delta$ , and can compare its price impact with that of other types of divestment, including in particular the sale of low-cost assets. Under the more general assumptions, the optimal divestment still includes capacity that the dominant firm withholds pre-divestment, and that is price-setting post-divestment. Furthermore, virtual

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<sup>17</sup>A virtual divestment that is explicitly designed to mimic the optimal divestment (i.e. with a maximum strike price equal to  $\hat{c}$ , and a strike price function with the same slope as the marginal cost function of the dominant firm) is actually equivalent to a forward contract, and therefore remains less effective than the optimal divestment. This follows from the fact that  $\hat{c} < p^* - \Delta_p$ .



divestments remain equivalent to the outright sale of low-cost assets. These analytical results are summarised in Appendix A.7.<sup>18</sup>

Appendix A.8 illustrates the results of the more general model by means of a simple numerical example. This example confirms that the main results of the analytical model are robust to variations in the cost parameters of the competing firms and in the demand slope parameter. In particular, the optimal divestment is still intermediate with respect to its location on the cost function of the dominant firm. Comparative statics can also be derived by means of the numerical example. In particular, we observe that as the costs of the dominant firm increase relative to the fringe or the demand slope parameter increases, the position of the optimal divestment converges to the cost of the most competitive capacity of size  $\delta$  withheld by the dominant firm in the pre-divestment equilibrium.

## 6 Conclusion

This paper has studied the impact of remedy design in a model where a dominant producer faces a competitive fringe. We analysed the effect on market prices, efficiency and profits of transfers of capacity from a dominant producer to a competitive fringe. We show that divesting capacity with intermediate costs can be several-fold more effective in reducing prices than an equivalent transfer of low-cost assets. In order to maximise the effectiveness of the divestment in reducing prices, the divested capacity needs to include assets which are sufficiently competitive to impose a competitive constraint on the dominant firm but whose costs are not too low so as to induce the dominant producer to accept a larger loss of its output post-divestment in order to keep prices high. In the optimal post-divestment equilibrium, the cost range of the divested capacity needs to span the post-divestment price (implying that some but not all of the divested capacity produces in equilibrium). We also find that the optimal divestment from the perspective of consumer welfare is always efficiency-increasing, and that it coincides with the socially optimal divestment for sufficiently large divestments.

We have also compared the effectiveness of outright asset sales to that of virtual divestments. We have established that the effectiveness of virtual divestments is maximised when all of the options that are sold are exercised, so that the remedy acts like a forward contract. Even if this is the case the virtual divestment reduces prices only as much as a physical divestment of low-cost capacity. Our findings therefore imply that outright divestments can be significantly more pro-competitive than their virtual equivalent. By the same token, both the dominant firm and the competitive fringe benefit from the imposition of a virtual divestment relative to a more effective outright divestment.

Our findings have direct policy relevance, given that divestments are frequently accepted by competition authorities as commitments in competition cases. Virtual divestments are also commonly used in the wholesale electricity sector. Our results are also applicable to the evaluation of merger effects, since divestments are the opposite of a merger. The findings of this paper imply that a merger where a larger incumbent buys strategic capacity from a smaller competitor can have significantly greater effect on prices than one where additional efficient capacity is purchased instead. Similarly, our results indicate that the price-increasing effect of the acquisition of a given volume of low-cost capacity by a firm with market power can be remedied by significantly smaller divestments of price-setting capacity.

A possible extension of the approach presented in this paper includes the analysis of the case

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<sup>18</sup>The proof of these results is available from the authors.

of asset divestments to new entrants or players with insignificant market power in the context of oligopoly interaction between incumbent firms. Obtaining comprehensive analytical results on the impact of the divestments of different type of assets in standard oligopoly models for homogenous products (e.g. Cournot or Supply Function Equilibria) is challenging, given the discontinuities created by asymmetric asset sales. However, some of the core results presented in this paper (in particular the fact that the divestment of low-cost assets is ineffective relative to the sale of assets with higher costs, and that virtual divestments are at best equivalent to the divestment of low-cost assets) can also be extended to oligopoly models, given the general nature of the underlying cost and demand effects that we identify in this paper.

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## Appendix A

### A.1 The post-divestment price function

Table 1: The post-divestment price function

Segment	Price	Range of $\bar{c}$
I	$p^* - \Delta_p$	$\gamma\delta \leq \bar{c} < \frac{p^*}{2} + \Delta_p$
II	$\gamma\mu - \bar{c}$	$\frac{p^*}{2} + \Delta_p \leq \bar{c} < \frac{p^*}{2} + 2\Delta_p$
III	$p^* - 2\Delta_p$	$\frac{p^*}{2} + 2\Delta_p \leq \bar{c} < \hat{c}$
IV	$\frac{3}{8}(\gamma(\mu - \delta) + \bar{c})$	$\hat{c} \leq \bar{c} < \gamma(\frac{3}{5}\mu + \delta)$
V	$\bar{c} - \gamma\delta$	$\gamma(\frac{3}{5}\mu + \delta) \leq \bar{c} < p^* + 3\Delta_p$
VI	$p^*$	$\bar{c} \geq p^* + 3\Delta_p$

where  $p^* = \frac{2}{3}\gamma\mu$ ;  $\Delta_p \equiv \frac{\gamma\delta}{3}$ ; and  $\hat{c} \equiv \gamma\left(\frac{2\sqrt{6}}{3} - 1\right)(\mu - \delta)$ .

### A.2 Proof of Proposition 1

This Proposition assumes that  $\delta \in \left[\left(1 - \frac{12}{5\sqrt{6}}\right)\mu, \left(1 - \frac{2}{\sqrt{6}}\right)\mu\right]$ , which we denote as  $\delta \in (\delta^L, \delta^H]$ .

The dominant firm's post-divestment marginal cost is defined by the following two-step function:

$$c_d = \begin{cases} \gamma q_d & \text{if } q_d < q' - \delta \\ \gamma(q_d + \delta) & \text{if } q_d \geq q' - \delta \end{cases},$$

where the first step corresponds to the pre-divestment cost function, whilst the second step corresponds to the cost-increasing effect of divestments described in the main text.

The competitive fringe's post-divestment marginal cost function (which is equivalent to the inverse residual demand of the dominant firm) is defined by the following three-step function:

$$c_f = \begin{cases} \gamma(\mu - q_d - \delta) & \text{if } q_d < \mu - q' - \delta \\ \frac{\gamma}{2}(\mu + q' - q_d - \delta) & \text{if } \mu - q' - \delta \leq q_d \leq \mu - q' + \delta \\ \gamma(\mu - q_d) & \text{if } q_d > \mu - q' + \delta \end{cases}.$$

where the first step corresponds to the demand-shift effect described in the main text, the second to the demand-slope effect, and the third coincides with the pre-divestment case.

As the model is discontinuous we have to study the firm's maximization problem in each of the regions defined by  $q'$  (for a given  $\delta$ ), in order to derive the post-divestment price function and identify the optimal divestment. This proof proceeds in three parts. First, we identify the four interior solutions that exist to the firm's maximisation problem. A unique candidate equilibrium can exist inside each region of  $q'$  since the model is linear. Second, we study equilibria at the regions where the feasibility conditions for the interior solutions are not satisfied. Third, we identify the level of  $q'$  at which post-divestment prices are minimised. The various candidate equilibrium cases that we identify in this proof are illustrated in Figure 3.

## Interior solutions

**Case I (low-cost divestment):** in this region the dominant firm and the competitive fringe of firms produce, respectively, at a higher and lower marginal cost than in the pre-divestment case, i.e.,  $c_d = \gamma(q_d + \delta)$  and  $c_f = \gamma(\mu - q_d - \delta)$ . The dominant firm maximises  $\pi_d^I = p^I q_d^I - \frac{\gamma}{2}(q_d^I)^2 - \gamma\delta q_d^I$  with respect to  $q_d^I$  and subject to  $p^I = c_f$ , which yields  $q_d^I = q_d^* - \frac{2}{3}\delta$ . This implies that  $p^I = p^* - \frac{\gamma\delta}{3}$ . The two feasibility conditions for this equilibrium are  $q_d^I < \mu - q' - \delta$  and  $q_d^I \geq q' - \delta$ . These conditions reduce to the following expression:

$$\text{Case I: } p^I = p^* - \frac{\gamma\delta}{3} \text{ for } q' \leq \frac{\mu + \delta}{3}.$$

**Case III:** the dominant firm produces at the pre-divestment marginal cost, while the competitive fringe produces at a lower marginal cost, i.e.,  $c_d = \gamma q_d$  and  $c_f = \gamma(\mu - q_d - \delta)$ . Thus,  $p^{III} = \gamma(\mu - q_d - \delta)$  and  $\pi_d^{III} = p^{III} q_d^{III} - \frac{\gamma}{2}(q_d^{III})^2$ . The first-order condition yields  $q_d^{III} = q_d^* - \frac{\delta}{3}$ , implying that  $p^{III} = p^* - \frac{2\gamma\delta}{3}$ . The feasibility conditions are  $q_d^{III} < \mu - q' - \delta$  and  $q_d^{III} < q' - \delta$ , which boils down to the following expression:

$$\text{Case III: } p^{III} = p^* - \frac{2\gamma\delta}{3} \text{ for } \frac{\mu + 2\delta}{3} < q' < \frac{2(\mu - \delta)}{3}.$$

**Case IV:** the dominant firm produces at the pre-divestment marginal cost, while the competitive fringe produces at the flatter part of its marginal cost function, i.e.,  $c_d = \gamma q_d$  and  $c_f = \frac{\gamma}{2}(\mu + q' - q_d - \delta)$ . Thus,  $p^{IV} = \frac{\gamma}{2}(\mu + q' - q_d - \delta)$  and  $\pi_d^{IV} = p^{IV} q_d^{IV} - \frac{\gamma}{2}(q_d^{IV})^2$ . From the first-order condition we obtain  $q_d^{IV} = \frac{\mu + q' - \delta}{4}$ , and  $p^{IV} = \frac{3}{8}(\gamma(\mu - \delta) + \bar{c})$ . The feasibility conditions can be expressed as follows:

$$\text{Case IV: } p^{IV} = \frac{3}{8}(\gamma(\mu - \delta) + \bar{c}) \text{ for } \begin{cases} \frac{3}{5}(\mu - \delta) \leq q' \leq \frac{3}{5}\mu + \delta & \text{if } \delta \leq \frac{\mu}{6} \\ \delta + \frac{\mu}{3} < q' \leq \frac{3}{5}\mu + \delta & \text{if } \delta > \frac{\mu}{6} \end{cases}.$$

**Case VI:** the dominant firm and the competitive fringe of firms produce at the pre-divestment marginal cost, so that the equilibrium is the same as the pre-divestment outcome. Here, the feasibility conditions imply the following:

$$\text{Case VI: } p^{VI} = p^* \text{ for } q' > \frac{2}{3}\mu + \delta.$$

**Overlap between Case III and Case IV.** Cases III and IV always overlap since  $\delta + \frac{\mu}{3} < \frac{2}{3}(\mu - \delta)$  for  $\delta < \frac{\mu}{5}$  (which is the case in the range of  $\delta$  that we consider), and  $\frac{3}{5}(\mu - \delta) < \frac{2}{3}(\mu - \delta)$  (which define the respective regions for  $\delta \leq \frac{\mu}{6}$ ). The overlap between these two interior solutions occurs because the marginal revenue schedule of the dominant firm jumps upwards at the first kink in the post-divestment residual demand curve, crossing the marginal cost schedule twice. In the overlap region the dominant firm will choose the equilibrium associated with the highest profits. From the expressions for profits derived above note that  $\pi_d^{III}$  does not depend on the value of  $q'$  whilst  $\pi_d^{IV}$  is increasing in  $q'$  (as we show formally below). We can therefore identify an indifference value for  $q'$  (defined as  $\hat{q}$ ) such that, for  $q' < \hat{q}$  we have  $\pi_d^{III} > \pi_d^{IV}$  (i.e. Case III represents the equilibrium outcome), and for  $q' > \hat{q}$  Case IV is the equilibrium.

From the equilibrium prices and quantities given above, we can compute that  $\pi_d^{III} = \frac{\gamma}{6}(\mu - \delta)^2$ , and  $\pi_d^{IV} = \frac{\gamma}{16}(\mu - \delta + q')^2$ . The profit-indifference point  $\hat{q}$  is therefore given by the following

quadratic condition (obtained by simplifying the two expressions for profits):  $\frac{(\mu-\delta)^2}{3} = \frac{(\mu-\delta+q')^2}{8}$ . This yields a unique positive root in  $q'$  given by:

$$q' = \left( \frac{2\sqrt{6}}{3} - 1 \right) (\mu - \delta) \approx 0.63 (\mu - \delta).$$

We therefore define  $\hat{q} \equiv \left( \frac{2\sqrt{6}}{3} - 1 \right) (\mu - \delta)$ . As we discuss in the main text, this identifies the optimal divestment policy for the range of  $\delta$  considered in this Proposition. Note that  $\bar{c}(\hat{q}) = \gamma\hat{q} \equiv \hat{c}$  is below  $p^*$ , as should be expected. We also have that  $\hat{c} > p^c$  (as stated in the Proposition) as long as  $\delta < \frac{4\sqrt{6}-9}{4\sqrt{6}-6}\mu$ , which is satisfied in the relevant range of  $\delta$ .

For  $q' = \hat{q}$  to be consistent with Case *IV* it needs to be contained within the range of  $q'$  that defines Case *IV*. This implies the following two conditions on  $\delta$ :

$$\begin{aligned} \hat{q} < \frac{3}{5}\mu + \delta &\Rightarrow \delta > \left( 1 - \frac{12}{5\sqrt{6}} \right) \mu \equiv \delta^L, \\ \hat{q} \geq \frac{\mu}{3} + \delta \text{ (for } \delta > \frac{\mu}{6}) &\Rightarrow \delta \leq \left( 1 - \frac{2}{\sqrt{6}} \right) \mu \equiv \delta^H. \end{aligned}$$

The two resulting conditions on  $\delta$  are assumed to hold in this Proposition, and define the range of intermediate divestments.

### Corner solutions

**Case II.** For  $\frac{\mu+\delta}{3} \leq q' < \frac{\mu+2\delta}{3}$  the interior solutions identified in Cases *I*, *III*, *IV* and *VI* are not feasible. In this region of  $q'$  we have that the first segment of the marginal revenue curve passes through the jump of the marginal cost curve, at  $q_d = q' - \delta$ . The dominant firm does not have incentive to produce more or less than  $q' - \delta$ , since the corresponding interior equilibria are not feasible. Thus, in this region the dominant firm finds it optimal to set  $q_d^{II} = q' - \delta$ , implying that  $p^{II} = \gamma(\mu - q')$ . We therefore have:

$$\text{Case II: } p^{II} = \gamma(\mu - q') \text{ for } \frac{\mu + \delta}{3} \leq q' < \frac{\mu + 2\delta}{3}.$$

**Case V.** For  $\frac{3}{5}\mu + \delta < q' < \frac{2}{3}\mu + \delta$  we have that the interior solutions of Cases *I*, *IV* and *VI* are not feasible, and that Case *III* is feasible only for  $\delta < \frac{\mu}{25}$ .

- We consider first the case where  $\delta \geq \frac{\mu}{25}$ . In this case, none of the interior solutions are feasible, because the marginal cost curve crosses the second (downwards) jump of the marginal revenue curve. It is therefore optimal for the dominant firm to price at the second kink of its residual demand curve, setting  $p^V = \gamma(q' - \delta) = \bar{c} - \gamma\delta$ . This is an equilibrium as long as  $p^V < p^{VI}$  (i.e.  $q' < \frac{2}{3}\mu + \delta$ ).
- If  $\delta < \frac{\mu}{25}$ , Cases *III* and *V* overlap (for  $\frac{3}{5}\mu + \delta < q' < \frac{2}{3}(\mu - \delta)$ ). If this is the case we have that profits in Case *V* are higher than those in Case *III*, and Case *V* therefore represents the post-divestment equilibrium. This follows from the fact that at the lower bound of the relevant range of  $q'$  (i.e.  $q' = \frac{3}{5}\mu + \delta$ ), we have that profits in Cases *IV* and *V* are equal (since the two cases are the same) and that profits in Case *IV* are higher than those in Case

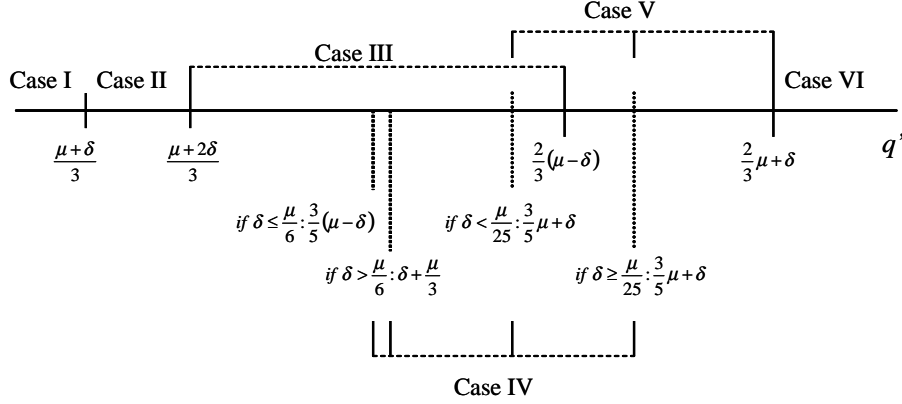


Figure 3: Summary of segments of the post-divestment price function.

*III* (since  $\frac{3}{5}\mu + \delta > \hat{q}$ ). We also have that  $\pi_d^V$  increases with  $q'$  (because higher values of  $q'$  imply that the kink on the residual demand curve on which the dominant firm is pricing is associated with a higher price level). Since  $\pi_d^{III}$  is constant in  $q'$ , it follows that  $\pi_d^V > \pi_d^{III}$  for  $\frac{3}{5}\mu + \delta < q' < \frac{2}{3}(\mu - \delta)$ .

We therefore have:

$$\text{Case V: } p^V = \bar{c} - \gamma\delta \text{ for } \frac{3}{5}\mu + \delta < q' < \frac{2}{3}\mu + \delta.$$

### Price comparison

The minimum  $p^{II}$  is achieved at  $q' = \frac{\mu + 2\delta}{3}$ , where  $p^{II} = p^* - \frac{2}{3}\gamma\delta = p^{III}$ . Therefore, for any level of  $\delta$ , we have the following price ranking:  $p^{III} = \min p^{II} < p^I < p^{VI} = p^*$ . The minimum  $p^V$  is achieved at  $q' = \frac{3}{5}\mu + \delta$ , where  $p^V = \frac{3}{5}\gamma\mu$ . The price corresponding to Case *IV* equals the minimum  $p^V$  for  $q' = \frac{3}{5}\mu + \delta$ , and takes a lower value for  $q' < \frac{3}{5}\mu + \delta$  (since it is increasing in  $q'$ ). Note also that the price set by the dominant firm at the point where the profits associated to Cases *III* and *IV* are equal is necessarily lower in Case *IV* than in Case *III*. The reason is that when the firm deviates to Case *IV* it prices on the flat part of its residual demand curve, moving away from pricing on the segment of the residual demand curve located to the left of the first kink. In order to do so it must expand output. From the above it follows that the lowest price is achieved when the divestment is located at the indifference point between Case *III* and Case *IV*, that is  $\hat{c} = \gamma\hat{q} \equiv \gamma \left( \frac{2\sqrt{6}}{3} - 1 \right) (\mu - \delta)$ .

The price at this point can be computed as  $p(\hat{c}) = \frac{\sqrt{6}}{4}\gamma(\mu - \delta)$ . The output of the dominant firm at the optimal divestment is given by  $q_d^{IV} = \frac{\mu + \hat{q} - \delta}{4} = \frac{\mu - \delta}{\sqrt{6}} \geq \frac{\mu}{3}$ .

### A.3 Proof of Proposition 2

This proof relies on the fact that the change in welfare from a divestment can be expressed as the change of total production costs due to the three output effects described in the main text: (1) the reduction in output of high-cost capacity owned by the fringe; (2) the change in output of the

divested capacity; and (3) the change in the output of the dominant firm net of the pre-divestment output of the divested assets. We denote as  $\Delta W^j$  the change in welfare obtained in each range  $j$  of the position of the divested capacity, as defined by the post-divestment price function, and  $\overline{\Delta W}^j$  as the maximal change in any given range. To compute welfare, we also rely on the post-divestment equilibrium price and output levels obtained in the proof of Proposition 1.

**Case I.** In this case only the first two output effects described above apply, and we have that

$$\Delta W^I = \int_{\frac{2\mu-\delta}{3}}^{\frac{2}{3}\mu} \gamma x dx - \int_{\frac{\mu}{3}}^{\frac{\mu+\delta}{3}} \gamma x dx = \frac{\gamma\delta}{9} (\mu - \delta).$$

**Case II.** As in the previous case, only the first two output effects apply, so that we have  $\Delta W^{II} = \int_{\mu-q'}^{\frac{2}{3}\mu} \gamma x dx - \int_{\frac{\mu}{3}}^{q'} \gamma x dx = \frac{\gamma}{2} \left( -\frac{4}{9}\mu^2 - 2(q')^2 + 2\mu q' \right)$ . Notice that  $\partial\Delta W^{II}/\partial q' > 0$  provided that  $\delta < \frac{\mu}{4}$ . Therefore welfare is maximised at the upper boundary of segment II (i.e.  $q' = \frac{\mu+2\delta}{3}$ ), where we have that  $\overline{\Delta W}^{II} = \frac{2\gamma\delta}{9} (\mu - 2\delta)$  (where the upper bar indicates the maximum welfare achieved in a given case).

**Case III.** All three output effects are relevant here, and we have that  $\Delta W^{III} = \int_{\frac{2}{3}(\mu-\delta)}^{\frac{2}{3}\mu} \gamma x dx + \int_{\frac{\mu}{3}}^{\frac{\mu}{3}} \gamma x dx - \int_{q'-\delta}^{q'} \gamma x dx = \gamma\delta \left( \frac{2}{9}\delta + \frac{5}{9}\mu - q' \right)$ . Hence  $\partial\Delta W^{III}/\partial q' < 0$ , so that total welfare is maximised at the lower boundary of the relevant range of  $q'$  (i.e.  $q' = \frac{\mu+2\delta}{3}$ ), where  $\overline{\Delta W}^{III} = \overline{\Delta W}^{II} = \frac{2\gamma\delta}{9} (\mu - 2\delta)$ .

**Case IV.** All three output effects also apply in this case, and we have that:

$$\begin{aligned} \Delta W^{IV} &= \int_{\frac{3}{8}(\mu-\delta+q')}^{\frac{2}{3}\mu} \gamma x dx - \int_{q'-\delta}^{\frac{3}{8}(\mu-\delta+q')} \gamma x dx - \int_{\frac{\mu}{3}}^{\frac{\mu-\delta+q'}{4}} \gamma x dx \\ &= \frac{\gamma}{2} \left( \frac{61}{288}\mu^2 + \frac{11}{16}\mu\delta - \frac{11}{16}\mu q' + \frac{21}{32}\delta^2 - \frac{21}{16}\delta q' + \frac{21}{32}(q')^2 \right). \end{aligned}$$

This expression shows that  $\partial\Delta W^{IV}/\partial q'$  is linear in  $q'$  but that its sign is ambiguous (i.e.  $\partial\Delta W^{IV}/\partial q' < 0$  only if  $q' < \frac{11}{21}\mu + \delta$ ). Therefore, there exist two possible efficiency-maximising points: one located at its lower boundary  $q' = \left( \frac{2\sqrt{6}}{3} - 1 \right) (\mu - \delta)$ ; and one at its upper boundary  $q' = \frac{3}{5}\mu + \delta$ .

**Case V.** In this case we have that only the first and third output effect apply, since the divested capacity does not produce in equilibrium. We therefore have that:  $\Delta W^V = -\int_{\frac{\mu}{3}}^{\mu-q'+\delta} \gamma x dx + \int_{q'-\delta}^{\frac{2}{3}\mu} \gamma x dx = \frac{\gamma}{2} \left( -\frac{4}{9}\mu^2 - 2(q')^2 - 2\delta^2 + 4q'\delta - 2\mu\delta + 2\mu q' \right)$ .

Notice that  $\partial\Delta W/\partial q' < 0$  provided that  $q' > \delta + \frac{\mu}{2}$ , which holds in Segment V. Therefore, the efficiency-maximising location is at  $q' = \delta + \frac{3}{5}\mu$ , which in turn implies  $\overline{\Delta W}^V = \frac{4}{225}\gamma\mu^2 = \Delta W^{IV}|_{q'=\delta+\frac{3}{5}\mu}$ .

### Socially optimal divestment

First, notice that  $\overline{\Delta W}^{II} = \overline{\Delta W}^{III} > \Delta W^I$  provided that  $\delta < \frac{\mu}{5}$ . Let us now denote by  $\Gamma^{j-k}$  the difference between  $\overline{\Delta W}^j$  and  $\overline{\Delta W}^k$ . We have that  $\Gamma^{III-V}(\delta) = \gamma \left( -\frac{4}{225}\mu^2 + \frac{2}{9}\mu\delta - \frac{4}{9}\delta^2 \right)$ . This implies that  $\Gamma^{III-V} = 0$  for  $\delta = \frac{2}{5}\mu$  (which is outside the range that we consider) and for  $\delta = \frac{\mu}{10}$ . It follows that  $\overline{\Delta W}^{III} < \overline{\Delta W}^V$  for  $\delta < \frac{\mu}{10}$ , given the concavity of  $\Gamma^{III-V}$  in  $\delta$ .



To complete the proof we need to compare the highest change of welfare achieved in either case *III* or *V* with the one obtained under case *IV* for  $q' = \left(\frac{2\sqrt{6}}{3} - 1\right)(\mu - \delta)$  (which is one of the two possible maxima for case *IV*, the other being equivalent to case *V*).

Consider first the case  $\delta < \frac{\mu}{10}$ . Here we must compare case *V* with case *IV*. We have:  $\Gamma^{IV-V}(\delta) = \gamma \left[ \left(\frac{327}{200} - \frac{2}{3}\sqrt{6}\right)\mu^2 + \frac{7}{8}\delta^2 - \left(\frac{7}{4} - \frac{2}{3}\sqrt{6}\right)\delta\mu \right]$ . This expression is convex in  $\delta$  and it equals 0 for  $\delta = \left(1 - \frac{2}{3}\sqrt{6}\right)\mu \equiv \delta^L$  and for  $\delta = \left(1 - \frac{38}{105}\right)\mu$  (which is above the relevant range of  $\delta$ ). We therefore have that  $\overline{\Delta W}^{IV} < \overline{\Delta W}^V$ , for  $\delta \in (\delta^L, \frac{\mu}{10})$ .

Consider now the case  $\delta \geq \frac{\mu}{10}$ . We must compare case *III* with case *IV*. Here we have that  $\Gamma^{IV-III}(\delta) = \gamma \left[ \left(\frac{119}{72} - \frac{2}{3}\sqrt{6}\right)\mu^2 + \frac{95}{72}\delta^2 - \left(\frac{71}{36} - \frac{2}{3}\sqrt{6}\right)\delta\mu \right]$ , which is also convex in  $\delta$ . From this we obtain that  $\Gamma^{IV-III} = 0$  for

$$\delta = \delta^W \equiv \frac{-24\sqrt{6} + 71 + 6\sqrt{-78 + 32\sqrt{6}}}{95}\mu \text{ and } \delta = \underline{\delta} \equiv \frac{-24\sqrt{6} + 71 - 6\sqrt{-78 + 32\sqrt{6}}}{95}\mu,$$

where  $\underline{\delta} < \frac{\mu}{10}$ . Therefore, for  $\delta \in (\frac{\mu}{10}, \delta^W)$ , we have that  $\Gamma^{IV-III} < 0$ , i.e., the welfare maximum corresponds to the maximum achieved under case *III*. Conversely, for  $\delta \geq \delta^W$ , we have that  $\overline{\Delta W}^{IV} > \overline{\Delta W}^{III}$ , as stated in the Proposition.

#### A.4 Corollary 1

Divestments that reduce market prices also reduce total industry profits. Assuming that the fringe purchases the divested capacity through a competitive process, then it makes no profits on the divested assets (as these profits are simply transferred to the dominant firm), and instead suffers a profit loss on the rest of its capacity, due to the reduction in infra-marginal rents and the loss of marginal output (net of the output produced by the divested assets). The loss suffered by the fringe is proportional to the impact of divestments on price. The dominant firm too loses out from a divestment, by construction (otherwise it could replicate the post-divestment outcome also pre-divestment).

Aggregate profits are obviously maximised in region *VI*, for  $q' \geq \frac{2}{3}\mu + \delta$ . If a more competitive divestment is chosen instead, producers will prefer a divestment located in either region *I* or *V*. In these two regions, both of the dominant firm's and the fringe's profit levels are positively correlated with prices. In the case of the dominant firm this follows from the fact that in region *I* the loss in profits is due to the reduction in price associated with an increase in the *combined* output of the dominant firm and the divested capacity of  $\frac{\delta}{3}$ , along the pre-divestment cost curve. In region *V*, the dominant firm expands output in order to reduce prices, and the divested assets do not produce. This has the same qualitative impact on the profits of the dominant firm (including those earned on the divested assets) as that of region *I*, with the only difference being the relative impact on prices and volumes. The dominant firm will therefore prefer the region associated with the smaller deviation from the pre-divestment optimum. The Proof of Proposition 1 further implies that the dominant firm's profits in the other possible regions of the post-divestment price function (i.e. segments *II*, *III* and *IV*) are lower than those in regions *I* and *V*, also accounting for the profits earned on the divested capacity (which are maximised in segment *I*).

These results mean that producers will individually and jointly prefer the type of divestment associated with a higher price. We have that  $p^I > p^V$  if  $q' < \frac{2}{3}(\mu + \delta)$ , so that producers will prefer a divestment in region *I* for  $q' < \frac{2}{3}(\mu + \delta)$ , and in region *V* otherwise.

## A.5 Proof of Proposition 3

Assume first that all options in the virtual divestment are exercised. In this case, the profit of the dominant firm is  $\pi_d = pq_d - \frac{\gamma}{2}(q_d)^2 + z - p\delta$ , where  $z = \sum_{j=1}^n f_j \delta_j$ . The first-order condition yields  $q_d = \frac{\mu + \delta}{3}$ , and therefore  $p = p^* - \frac{\gamma\delta}{3}$  (where  $\Delta_p \equiv \frac{\gamma\delta}{3}$ ). The feasibility conditions are that  $p > f_n$  and  $q_d \geq \delta$ . The former implies:  $f_n < p^* - \frac{\gamma\delta}{3}$ . To satisfy the second condition we require  $\delta \leq \frac{\mu}{2}$  (which is within the range of intermediate divestments that we study).

To show that the largest price reduction achieved by a virtual divestment equals  $\Delta_p$ , we need to also consider the case of a set of contracts for which the dominant firm finds it optimal to set a price such that some options are not exercised. For a given price  $p$  such that  $f_x \leq p < f_n$ , the number of options that are exercised is defined as  $x(p)$ . Hence the dominant's profit is  $\pi_d = pq_d - \frac{\gamma}{2}(q_d)^2 + z(x(p)) - p\delta(x(p))$ , where  $z(x(p)) = \sum_{j=1}^{x(p)} f_j \delta_j$  and  $\delta(x(p)) = \sum_{j=1}^{x(p)} \delta_j$ . The optimal price for the dominant firm can be found by first finding the price that maximizes its profit function for each number of exercised options in isolation. And second, by comparing the profit in each of these cases. The optimal price will be given by the one that yields the maximum profit among all the possible cases.

Suppose that  $p$  is such that  $\bar{x}$  options are exercised, then by maximizing  $\pi_d(\bar{x}) = pq_d - \frac{\gamma}{2}(q_d)^2 + z(\bar{x}) - p\delta(\bar{x})$  we find that  $p(\bar{x}) = p^* - \frac{\gamma\delta(\bar{x})}{3}$  and  $q(\bar{x}) = \frac{\mu + \delta(\bar{x})}{3}$ , provided that  $f_{\bar{x}+1} > p(\bar{x}) \geq f_{\bar{x}}$ , otherwise  $p(\bar{x}) = f_{\bar{x}}$ . Therefore, for each  $x \in (1, \dots, n-1)$  we can construct a candidate equilibrium in which the dominant firm's profit is given by  $\pi_d(p(x), q(x))$ . Let  $y$  be such that  $\pi_d(p(y), q(y)) \geq \pi_d(p(x), q(x))$  holds for any  $x \neq y$ . It follows that  $p^\nu(\Theta, \delta) = p(y)$ . Notice that  $p(y) > p^* - \frac{\gamma\delta}{3}$  since  $\delta(y) < \delta$ , which completes the proof of the Proposition.

## A.6 Proof of Proposition 4

The results shown so far imply that welfare impact of the optimal virtual divestment is the same as the one of a low-cost divestment situated in segment  $I$ . It is therefore equal to  $\frac{\gamma\delta}{9}(\mu - \delta)$ , as shown in the Proof of Proposition 2. Recall also that at the optimal divestment, the price and output of the dominant firm are respectively given by  $p(\hat{c}) = \frac{\sqrt{6}}{4}\gamma(\mu - \delta)$  and  $q_d^{IV} = \frac{\mu - \delta}{\sqrt{6}}$ . These results imply that the welfare impact of the optimal divestment from the perspective of consumers (which we define as  $\Delta\hat{W}$ ) is given by the following expression:

$$\Delta\hat{W} = \int_{\frac{\sqrt{6}}{4}(\mu - \delta)}^{\frac{2}{3}\mu} \gamma x dx - \int_{\frac{\mu}{3}}^{\frac{\mu - \delta}{\sqrt{6}}} \gamma x dx - \int_{\left(\frac{2\sqrt{6}}{3} - 1\right)(\mu - \delta) - \delta}^{\frac{\sqrt{6}}{4}(\mu - \delta)} \gamma x dx = \gamma \left[ \left( \frac{119}{72} - \frac{2}{3}\sqrt{6} \right) \mu^2 + \frac{7}{8}\delta^2 - \left( \frac{7}{4} - \frac{2}{3}\sqrt{6} \right) \delta\mu \right].$$

We thus have that  $\Delta\hat{W} - \Delta W^I = \gamma \left[ \left( \frac{119}{72} - \frac{2}{3}\sqrt{6} \right) \mu^2 + \frac{71}{72}\delta^2 - \left( \frac{67}{36} - \frac{2}{3}\sqrt{6} \right) \delta\mu \right]$ . The gradient of  $\Delta\hat{W} - \Delta W^I$  is given by  $\nabla(\mu, \delta) = \left( \frac{d}{d\mu} \left( \Delta\hat{W} - \Delta W^I \right), \frac{d}{d\delta} \left( \Delta\hat{W} - \Delta W^I \right) \right)$ , with  $\nabla(0, 0) = 0$ . The Hessian matrix of  $\Delta\hat{W} - \Delta W^I$  is positive definite. Hence,  $\mu = \delta = 0$  is a global minimum, where  $\Delta\hat{W} - \Delta W^I = 0$ . For any positive  $\mu$  and  $\delta$ , we thus have that  $\Delta\hat{W} - \Delta W^I > 0$ .

## A.7 Summary of results in the general case

In the general case we assume that  $c_i = \gamma_i q_i$  for  $i = d, f$  and that  $q_d + q_f = \mu - \rho p$ . The residual demand curve faced by the dominant firm is therefore given by  $p = \frac{\mu - q_d}{x}$  where  $x \equiv \rho + \frac{1}{\gamma_f}$ . Maximisation of the dominant's firm profit function yields the pre-divestment price and output following straightforward calculations:

$$q_d^* = \frac{\mu}{2 + \gamma_d x}; \text{ and } p^* = \frac{1 + \gamma x \mu}{2 + \gamma x x}$$

The various segments of the post-divestment price function can be derived using the same methodology used in the baseline model, by identifying the various steps in the post-divestment cost and residual demand function of the dominant firm, and establishing the feasibility conditions for each solution to the optimisation problem. Using this methodology we obtain the following results for low-cost divestments (in segment *I* of the post-divestment price function, as defined in the baseline case), and at the optimal divestment (which is located in the overlap between segments *III* and *IV*, as in the baseline case). The corresponding proof for these results is available separately from the authors.

**Case I (low-cost and/or virtual divestment).** In this case we obtain that  $p^I = p^* - \frac{\delta}{x(2 + \gamma_d x)}$ , which simplifies to the baseline result for  $\gamma_d = \gamma$  and  $x = \frac{1}{\gamma}$ . This is the same price effect that is obtained by imposing a forward contract of size  $\delta$ , with a maximum strike price below  $p^I$ .

**Optimal divestment.** We find that the optimal divestment is located at the following point in the cost function of the dominant firm:  $\hat{c} = \gamma_d (\sqrt{A} - 1) (\mu - \delta)$ , where

$$A \equiv \frac{(3\gamma_f + \gamma_d + \rho\gamma_d\gamma_f)(\gamma_f + \gamma_d + \rho\gamma_f\gamma_d)}{(\gamma_d + 2\gamma_f + \rho\gamma_d\gamma_f)(1 + \rho\gamma_f)\gamma_d}.$$

Note that in the baseline case  $A$  simplifies to  $\frac{8}{3}$ , so that  $\sqrt{A} = \frac{2\sqrt{6}}{3}$ . We also have that the price at the optimal divestment equals  $p(\hat{c}) = \frac{\sqrt{A}\alpha(\alpha + \gamma_d)}{2\alpha + \gamma_d} (\mu - \delta)$ , where  $\alpha \equiv \frac{\gamma_d\gamma_f}{\gamma_d + \gamma_f + \gamma_d\gamma_f\rho}$ . For  $\gamma_d = \gamma_f = \gamma$  and  $\rho = 0$  we have that  $\alpha = \frac{\gamma}{2}$ , so that we obtain that  $p(\hat{c}) = \frac{\sqrt{6}}{4}\gamma(\mu - \delta)$ , confirming the result obtained for the baseline case.

We also have that the optimal divestment is feasible for  $\delta \leq \delta^H \equiv \left(1 - \frac{1}{\sqrt{A}} \left(\frac{2\alpha + \gamma_d}{\alpha + \gamma_d}\right)\right) \mu$ , which is the binding condition in the numerical example presented in the text (for sufficiently high values of  $\gamma_d$  or  $\rho$ , and sufficiently low values of  $\gamma_f$ ). This condition too collapses to the equivalent condition in the baseline case ( $\delta \leq \delta^H \equiv \left(1 - \frac{2}{\sqrt{6}}\right) \mu$ ) for  $A = \frac{8}{3}$  and  $\alpha = \frac{\gamma}{2}$ .

## A.8 Robustness of results: numerical example

In this example we assume that  $\mu = 90$ ,  $\delta = \frac{\mu}{10}$ , and (for the baseline case)  $\gamma_d = \gamma_f = 1$ . These assumptions imply that in the baseline case we have a pre-divestment price of 60, a price at the optimal divestment of 49.6 and a price following the divestment of low-cost assets (or a virtual divestment) of 57. The ratio of the price impact of the optimal divestment and of the divestment of low-cost assets (i.e.  $R$ ) therefore equals 3.47. We also have that the position of the optimal divestment in the baseline case equals 51.3, above both the competitive price of 45 and the corresponding post-divestment price of 49.6.

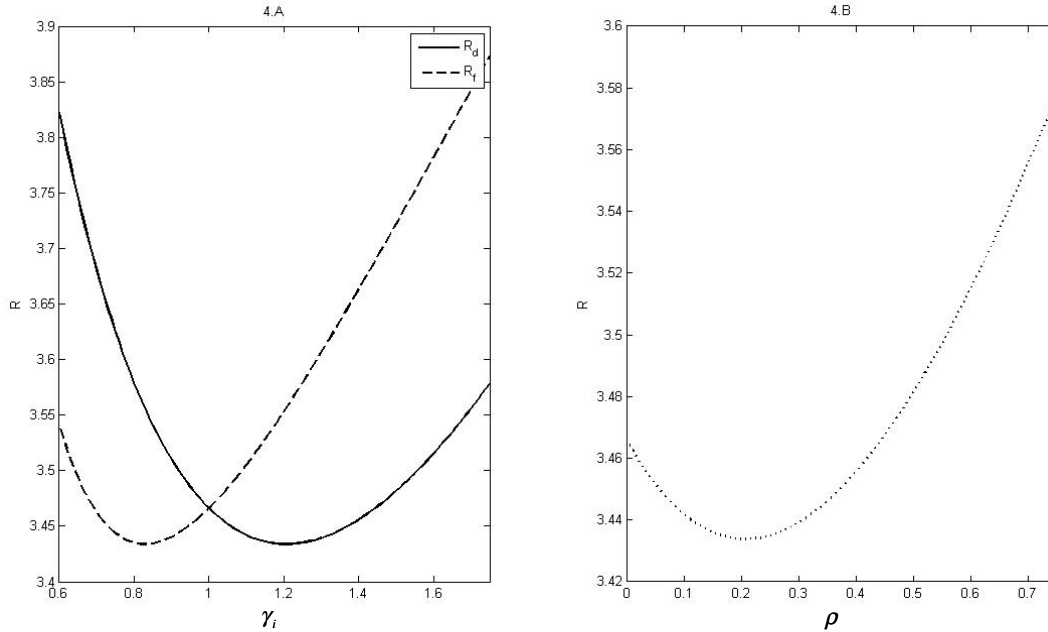


Figure 4: Values of  $R\left(\frac{\delta}{\mu}\right)$  as  $\gamma_d$ ,  $\gamma_f$  or  $\rho$  vary, respectively shown in Panel 4.A (continuous line ( $R_d$ )), Panel 4.A (dotted line ( $R_f$ )) and Panel 4.B.

Figure 4 illustrates the relative price impact function  $R$  as we respectively relax the assumption of cost-symmetry (allowing each of  $\gamma_d$  and  $\gamma_f$  to vary between 0.6 and 1.75, keeping the other cost parameter constant at 1); and the one of price-inelastic demand, allowing the demand slope parameter  $\rho$  to vary between 0 and 0.75.<sup>19</sup> Both panels of the figure illustrate the fact that in the more general model the optimal divestment remains much more effective than divestments of low-cost capacity in reducing prices. In the numerical example that we consider the value of the function  $R$  never falls below 3.4 as the cost parameter varies, and it increases as the degree of dominance rises through a reduction of the costs of the dominant firm relative to the competitive fringe (either because of a lower value for  $\gamma_d$  or a higher value for  $\gamma_f$ ). The value of  $R$  reaches a maximum of close to 3.9 if the cost of the fringe is at the upper bound of the range that we consider. The function  $R$  remains above a value of 3.4 also as the slope of the demand function increases, and it takes a higher value than in the baseline case for  $\rho$  sufficiently high.

Figure 5 plots our results on the position of the optimal divestment as the parameters of the model vary, relative to three possible benchmarks: the pre-divestment price, the competitive price, and the cost of the most competitive capacity of size  $\delta$  withheld by the dominant firm in the pre-divestment equilibrium (recall that this benchmark takes the value of  $\gamma\left(\frac{\mu}{3} + \delta\right)$  in the baseline model). For the baseline case we established the the highest cost of the optimal divestment takes an intermediate value that lies below the first benchmark, but above the other two. This result is confirmed in the more

<sup>19</sup>This parameter range is selected to ensure that the condition  $\delta \leq \delta^H$  is satisfied (so that we are in a range of intermediate divestments). In the baseline case considered in this numerical example we have that  $\delta^H = 16.51$  (which is well above the size of the divestment used for the illustration, i.e.  $\delta = \frac{\mu}{10} = 9$ ). In this example we also have that  $\delta = \delta^H = 9$  for the following parameter combinations:  $\{\gamma_d = 1.78; \gamma_f = 1; \rho = 0\}$ ;  $\{\gamma_d = 1; \gamma_f = 0.56; \rho = 0\}$ ;  $\{\gamma_d = \gamma_f = 1; \rho = 0.78\}$ .

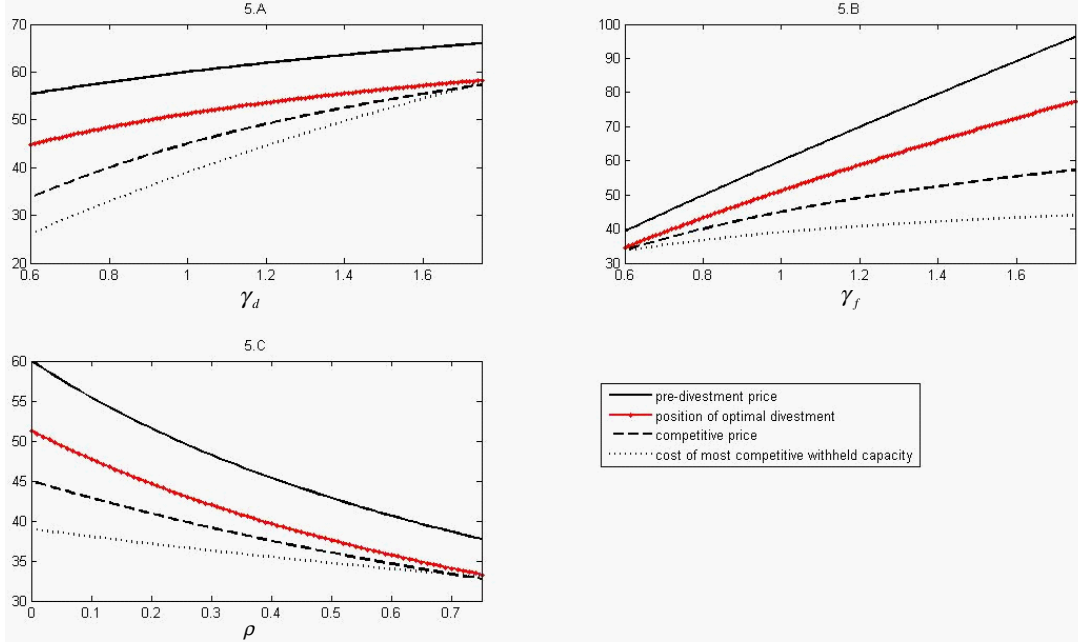


Figure 5: Relative position of the optimal divestment, as  $\gamma_d$ ,  $\gamma_f$  or  $\rho$  vary (respectively shown in Panels 5.A, 5.B and 5.C)

general model, as shown in the figure. We find that as the relative costs of the dominant firm increase relative to the fringe (so that its pre-divestment share of the market falls, and the effective size of the divestment increases towards  $\delta^H$ ), the position of the optimal divestment converges towards the two lower benchmarks described above. For  $\delta = \delta^H$  the location of the optimal divestment exactly equals the cost of the most competitive capacity withheld by the dominant firm in the pre-divestment equilibrium, which is the same result that we obtain in the baseline model. We obtain a similar result as demand becomes more elastic (i.e. when  $\rho$  increases), as it is shown in the corresponding panel of the figure.